Masconomet Regional High School Curriculum Guide

COURSE TITLE:	BC Calculus	**COURSE NUMBER:**	1441
DEPARTMENT:	Mathematics	**GRADE LEVEL(S) & PHASE:**	12, AP
LENGTH OF COURSE:	Full Year		

I. Course Description:

This course is a rigorous, college-level course in Calculus. It is equivalent to a one-semester college calculus sequence and follows, but is not limited to, the "BC course description" as published by the College Board and found at www.collegeboard.org/ap/calculus. The course is a balanced mixture of theoretical and intuitive approaches in the study of the various concepts. It is assumed that each student brings a wealth of mathematical knowledge and fluency to this course. Particularly necessary for success is a firm grasp of functions, both algebraic and transcendental. The course will prepare the student for the BC Level Advanced Placement Examination in Calculus.

II. Central Objectives:

A. To understand the concept of limit, including the evaluation of and the verification of each form of limit.
B. To understand the concept of the derivative using both the definition and the theorems to differentiate functions.
C. To solve problems utilizing an understanding of derivatives, including related rates, maxima/minima, curve sketching, and Newton's method.
D. To thoroughly understand the relationship between derivative, anti-derivative and definite integral as related in the Fundamental Theorem of Calculus.
E. To understand methods of numerical approximation of the definite integral, including Riemann, rectangular and trapezoidal.
F. Utilize integral calculus to solve application problems involving area, volume of solids, and lengths of curves.
G. Understand integrals as the accumulation of rates of change, particularly involving distance, velocity, and acceleration.
H. Utilize differential equations to model natural phenomena. Determine functions through numerical, graphical and analytical techniques.
I. To become familiar with the techniques of determining indefinite integrals, such as substitution and integration by parts.
J. To understand power series as they relate to geometric series, functions, integrals and derivatives. Be able to utilize techniques to determine intervals of convergence
K. Be able to apply the underlying principals of rates of change and area to parametric functions, polar functions, and vectors.

III: Text and Additional Material:

Primary Text:

Single Variable Calculus: Early Transcendentals
Rogawski, Jon: W.H. Freeman and Company: 2008

Back-up Text:

Calculus: Graphical, Numerical, Algebraic
Finney, Ross L., Franklin D. Demana, Bert K. Waits, and Daniel Kennedy: Addison Wesley Longman: 1999

A dedicated website - http://www.masconomet.org/teachers/rschonewald/calculus.htm allows students to access tutorial and homework help. In addition, some material is used to help students visualize and better understand concepts during class. For instance, there are links to some excellent sites related to rotating areas to form solids. Enrichment/post test activities include "Find the Error", an excellent source which challenges and deepens student's understanding of concepts.

IV Pace/Plan of Instruction/ Book / Ancillary material

Topic #1 Limits and Continuity — Text: Ch 2
- rates of change and limits — Time: 13 school days
-Limits involving infinity
-Continuity
-Rates of change and tangent lines
-delta/epsilon proofs

Student Activities

Day One: Students examine a quadratic function and its average rate of change. The delta x is gradually reduced so that the average slope approached the tangent line. Students see graphically the secant line and then numerically calculate the changing slope. Finally an analytic solution is derived by taking a limit. This sets the stage for why we talk about limits and continuity before derivatives.

Finishing exercise: A look at machining tolerances as related to d/e proofs. What impact do tolerances have on machining costs and how compromise is reached?

Topic #2 Derivatives — Text: Ch 1 and 3
- Derivative of a function — Time: 20 days
-rules for Differentiation
-Velocity and other rates of change
- Derivatives of Trig functions
-Intermediate value theorem for derivatives
-Chain Rule
-Implicit differentiation
- Derivatives of inverse functions
-derivatives of exponential and log functions

Student Activities

-A review of functions is inculcated in the discussion and work for derivatives of various functions.

-The exponential function derivative is introduced by asking the question "What would a function look like in which f '(x) = f(x)". Students are asked to draw it and explain why it works.

-Students are introduced to their graphing calculator derivative function by looking at the y=abs(x) function and the lie that their calculator tells them.

-Time permitting, mini-project: **Sunrise, sunset** – pick a city and fit a trig function to sunrise, sunset and sunlight times, take derivative and determine max and min rates of change.

Topic #3 Applications of Derivatives — Text: Ch 3 and 4
- Extreme values and related Thms. — Time: 15 days
- Mean value theorem
-Connecting f', f" with f
-Modeling and optimization
-Linearization and Newton's Method
-Related Rates

Student Activities

http://www.masconomet.org/teachers/rschonewald/bigrace.htm To finish the section on applications of derivatives, students complete the project "The Big Race" which requires the modeling of a complex situation, and finding a minimum by taking a derivative. Students who do work correctly will win the race.

Graphing calc shortcut is demonstrated for repeated Newton Method Approximations

Students spend a day, figuratively, at the Acme Packaging Co. as a packaging engineer, optimizing boxes and tin cans. Material is provided for students to "guess and check". An analytic solution is

eventually discussed and determined by the students. A related question is: Does the Extreme Value Thm apply in these cases?

Topic #4 The definite Integral — Text: Ch 5
- Estimating with Finite Sums — Time: 13 days
-Definite Integrals
-Definite Integrals and Antiderivatives
-Fundamental Theorem of Calculus
-Trapezoidal Rule

Student Activities

There are several effective explorations in the text for FTC. Students draw the same function but then start at a place of their own choosing and draw thin vertical lines to fill in the area under the curve. They then stop at a place of their choosing and we discuss the instantaneous rate of change of the area function

To reinforce the FTC concept, students take ten minutes to prepare and give an explanation to a colleague on why an antiderivative gives the area under the curve. The test in this section requires students to explain the proofs for the 1^{st} and 2^{nd} parts of the FTC

Topic #5 Differential Equations and Math. Modeling — Text: Parts of Ch 5, 6, and 7
- Antiderivatives and Slope fields — Time: 20 days
-Integration by Substitution
-Integration by parts
- Exponential growth (y'=ky) and decay
- Population Growth – logistic

Student Activities

- Application of integrals – Newton's Law of Cooling. Students work together on a lab to determine which cools faster, liquid in an open Styrofoam cup or a Diet Coke can. CBL Lab Pro's are utilized.

- Graphing calculator programs are utilized throughout the course. In the differential equations section, students have a slopefield program as well as an Euler's program. This allows them to approach differential equations from a graphical, numerical and analytical perspective. A series of differential equations are given and students inspect the slopefields, use Euler's method to obtain estimates of points on the graph and determine if an analytic solution is possible. In addition, material from a Deborah Hughes-Hallett seminar is utilized for a class discussion on the recent SARS outbreak. The slopefield for disease spread is examined and talked about. Harvard Magazine had a recent article in their March/April 2007 issue which will be read by the students next year. Students see how differential equations are applied in the real-world. After the AP test, for students that remain, they read "Tipping Point", and discuss connection with differential equations.

Topic #6 Midterm Review and Exam — Text: Ch 1- 7
Time: 10 days

- Students conduct the 1^{st} semester final review themselves. Working in groups, they prepare material for the section for which they are responsible, including theory and sample problems. They are given 15 minutes to review material in a presentation and are graded for clarity, accuracy and command of material.
-The midterm uses typical AP multiple choice questions and abbreviated open response questions.

Topic #7 Applications of Definite Integrals — Text: Ch 6 and 8
- Integral as Net Change — Time: 16 days

-Areas in the Plane
-Volumes
-Lengths of Curves

Student Activities

- CBL Motion detectors are used so that students can get a grasp of summing up rates of change. Students look at the velocity graphs and calculate displacement from them and compare to the actual displacement.
- Students bring in Play-doh and model volumes of revolution. Using dental floss, they perform various slices so as to familiarize themselves with the disk, washer and shell methods.
- Since it is difficult to visualize 3D geometry, a SmartBoard is used to show some excellent web sites that do the volume creation and analysis.

Topic #8 - L'Hopital's Rule, Improper Integrals, and Partial fractions

Text: Parts of Ch 4 and 7
Time: 6 days

- L'Hopital's Rule
- Improper Fractions
- Partial fractions and Integral Tables

Student Activities

Some time is spent on "Gabriel's Horn", a revolved shape that has an infinite surface area, but only a finite volume
Some theoretical holes in their Pre-calculus background are filled in by showing the theory behind the number "e" and the compounding continuously equation A= Pe^rt. Otherwise, it is a rather dry section

Topic #9 Infinite Series

Time: 22 days
Text: Ch 10

- Power Series
- Taylor Series
- Taylor's Theorem
- Radius of conversion
- Testing Convergence at Endpoints

Student Activities

An understanding of geometric series is reinforced before introducing power series. Showing that the infinite series for cos (x) converges for all x is a challenge to student's understanding and fosters discussion

Error estimates, particularly Taylor's thm tends to be confusing for students. Therefore an incremental approach is taken starting with the error associated with a geometric series, then convergent alternating series, followed by a graphing calculator assisted approach to determining error, followed by the Lagrange error bound. The methods are compared, with particular emphasis on the concept of bounding error. Students are also challenged by the concept that an alternating series that converges conditionally can be made to converge to any chosen value.

Understanding the various tests for convergence can be equally confusing. A finishing exercise is called "running the gauntlet" where 15 exercises are given. For the first five, students work in groups to determine which method(s) should be used for convergence, the next five in pairs, and the final five by themselves, with a prize to the person(s) who can get them all. The methods used are the nth term, Limit Comparison Test, Direct Comparison Test, Ratio Test, p-series and the Integral Test.

Topic #10 Parametric, Vector and Polar Functions

Time: 15 days
Text: Ch11 and alternate text (FDWK) for discussion of vectors

- Parametric Functions
- Vectors in the Plane
- Vector-Valued Functions
- Polar Coordinates and Polar Graphs
- Calculus of Polar Curves

Student Activities

Since students spend significant time on vectors in their physics class, they are expected to review vectors on their own. For a review and deeper understanding of polar graphs, students use their graphing calculators as a means to investigate various trends.

Topic #11 AP Test Preparation

Text: Barron's, material from previous AP tests, AP Central
Time: 14 days

- AP Review – Curriculum is completed by the end of the 3rd quarter. For students who decline to take the AP test and do a senior internship, they will have seen all of the material that could appear on the AP test. Review is done using several different resources. They consist of a review of concepts and typical multiple choice problems from Barron's How to Prepare for the AP Calculus Exam". In addition, released AP exams and open response questions from previous exams are worked. Students will have seen open response questions during the course of the year on exams and practice problems.

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